

# Nonlocal effect on the magnetic penetration depth in multigapped superconductors

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Nonlocal effect is of fundamental importance to the understanding of superconductivity, as it plays a key role in various physical phenomena observed in superconductors. In general, the term “nonlocal” bears a meaning that the electromagnetic response of superconductors is not described by local equations due to the fact that the Cooper pairs carrying the supercurrent have a finite distance of correlation (the “coherence length”,  $\xi_0$ ). Meanwhile, the same term often refers to the situation that the physical quantities controlling superconductivity depends on electronic momenta. In this short note, we discuss the nonlocal effect in the former sense, where it is pointed out that the effective magnetic penetration depth ( $\lambda$ ) in the multigapped superconductors may exhibit decrease with decreasing external field ( $B_0$ ) in a manner  $\lambda \propto \sqrt{B_0/B_{c2}^*}$  (with  $B_{c2}^*$  being the upper critical field for the smallest energy gap) due to the nonlocal effect.

In type I superconductors, it is well established that the nonlocal effect leads to the enhancement of the penetration depth,

$$\lambda \simeq (\lambda_L^2 \xi_0)^{1/3}, \quad (1)$$

where  $\lambda_L$  corresponds to the local London penetration depth.<sup>1</sup> The above estimation is obtained from the elementary argument for a bulk superconductor, considering a mean vector potential ( $\bar{A} = \lambda B_0$ ) over a surface layer of thickness  $\lambda$  (which is zero elsewhere). Here, let us follow the similar line to clarify the situation in type II superconductors. We introduce a vector potential for an isolated flux line.

$$A(r) = \frac{\Phi_0}{2\pi r}, \quad (2)$$

where  $\Phi_0$  is the flux quantum and  $r$  is the distance from the center of the flux. The above equation may be approximated into a mean potential,

$$\bar{A} \simeq \frac{\Phi_0}{2\pi\lambda} \simeq \frac{SB_0}{2\pi\lambda} \quad (3)$$

within a cylinder ( $0 \ll r \leq \lambda$ ) and zero elsewhere (with  $S$  being the unit surface of flux line lattice). The average current density  $\bar{J}$  around the flux can be estimated by using the nonlocal form of the supercurrent and  $\bar{A}$ ,

$$\bar{J} \simeq -\frac{c}{4\pi\lambda_L^2} \frac{\lambda^2}{\xi_0^2} \bar{A} = -\frac{c\lambda}{4\pi\lambda_L^2} \frac{SB_0}{2\pi\xi_0^2}, \quad (4)$$

where the factor  $\lambda^2/\xi_0^2$  comes from the ratio of volume for the region of non-zero  $\bar{A}$  ( $\propto \pi\lambda^2$ ) to the effective integration volume ( $\propto \pi\xi_0^2$ ). Combining eq. (4) with a Maxwell equation around the flux, we have

$$\frac{B_0}{\lambda} \simeq |\langle \text{curl} \mathbf{B} \rangle| = \frac{4\pi\bar{J}}{c} \simeq \frac{\lambda}{\lambda_L^2} \cdot \frac{SB_0}{2\pi\xi_0^2}, \quad (5)$$

from which we obtain a solution

$$\lambda \simeq \lambda_L \sqrt{\frac{2\pi\xi_0^2}{S}}. \quad (6)$$

Using an expression for the upper critical field

$$B_{c2} = \frac{\Phi_0}{2\pi\xi_0^2} = \frac{SB_0}{2\pi\xi_0^2}, \quad (7)$$

eq. (6) is rewritten as

$$\lambda \simeq \lambda_L \sqrt{\frac{B_0}{B_{c2}}}. \quad (8)$$

Thus, when the nonlocal effect is significant (i.e.,  $\lambda_L \leq \xi_0$ ), the penetration depth around flux line is renormalized by a factor  $\sqrt{B_0/B_{c2}}$ . Interestingly, the effect discussed here serves to *reduce* the penetration depth, which is in marked contrast to the case of eq. (1).

It would be needless to mention that the situation considered in the above estimation cannot be realized in ordinary type II superconductors with a single gap, because the condition of  $\lambda_L \leq \xi_0$  is never satisfied. However, in the case of multigapped superconductors, it might happen that the coherence length corresponding to the smaller gap may be comparable to the penetration depth. To examine the practical situation, let us discuss an order parameter having two energy gaps,  $\Delta_i$  ( $i = \sigma, \pi$ ) with  $\Delta_\pi < \Delta_\sigma$ , that is actually realized in MgB<sub>2</sub>.<sup>2</sup> The presence of two gaps implies that there are two corresponding coherence lengths and associated upper critical fields

$$B_{c2(i)} = \frac{\Phi_0}{2\pi\xi_i^2}, \quad (9)$$

$$\xi_i \simeq \frac{\hbar v_F}{\pi\Delta_i} \quad (i = \sigma, \pi), \quad (10)$$

with  $v_F$  being the Fermi velocity. Provided that  $\lambda_L \leq \xi_\pi$ , the nonlocal effect leads to the renormalization of the penetration depth into an effective one,

$$\lambda \simeq \lambda_L \sqrt{b_\pi}, \quad (11)$$

where  $b_\pi \equiv B_0/B_{c2(\pi)}$ . It must be noted that superconductivity is maintained by the larger gap ( $\Delta_\sigma$ ) for  $B_0 < B_{c2(\sigma)}$ , while  $\Delta_\pi$  collapses and associated nonlocal effect disappears when  $B_0 > B_{c2(\pi)}$ . Meanwhile, eq. (11) would not be valid for  $\lambda < \xi_\sigma$ , where  $J$  is reduced to zero. Thus, the nonlocal effect due to the smaller gap is predicted to occur only over the lower field range  $\xi_\sigma^2 B_{c2(\pi)}/\lambda_L^2 \leq B_0 < B_{c2(\pi)}$  (or  $\xi_\sigma^2/\lambda_L^2 \leq b_\pi < 1$ ).

It is well established that muon spin rotation ( $\mu$ SR) measurement in type II superconductors provides a direct information on the spatial distribution of magnetic field

$$B(\mathbf{r}) = \sum_{\mathbf{K}} b(\mathbf{K}) \exp(-i\mathbf{K} \cdot \mathbf{r}), \quad (12)$$

where  $\mathbf{K}$  are the vortex reciprocal lattice vectors. The Fourier component of the field profile,  $b(\mathbf{K})$ , is determined by the penetration depth and coherence length; if we adopt the London model, we have

$$b(\mathbf{K}) = \frac{B_0 \exp(-\frac{1}{2}\xi_c^2 K^2)}{1 + \lambda^2 K^2}, \quad (13)$$

where  $\xi_c$  ( $\propto \xi_0$ ) is the cutoff parameter to correct the nonlocal electrodynamics near the vortex cores. In a Gaussian approximation, the spin relaxation observed by  $\mu$ SR is described by the Gaussian decay with a relaxation rate

$$\sigma_\mu = \gamma_\mu \langle \sum_{\mathbf{K}} b(\mathbf{K})^2 \rangle^{1/2} \simeq \frac{G(b)}{\lambda^2}, \quad (b \equiv B_0/B_{c2}) \quad (14)$$

in which  $\gamma_\mu$  is the muon gyromagnetic ratio, and the factor  $G(b) \propto (1-b)[1+3.9(1-b)^2]^{1/2}$  represents the reduction of  $\sigma_\mu$  mainly due to the stronger overlap of vortices ( $\propto 1-b$ ) and additional decrease due to the contribution of vortex cores at higher fields.<sup>3</sup> The nonlocal effect for the two energy gaps may be incorporated by substituting Eq. (6) to the above equations so that the relaxation rate exhibits a field dependence

$$\sigma_\mu \simeq \begin{cases} G(b)/(\lambda_L^2 b_\pi), & \xi_\sigma^2/\lambda_L^2 \leq b_\pi < 1 \\ G(b)/\lambda_L^2, & 1 < b_\pi < B_{c2(\sigma)}/B_{c2(\pi)} \end{cases} \quad (15)$$

where  $b = B_0/B_{c2(\sigma)}$ . It implies that  $\sigma_\mu$  may exhibit strong deviation from that for the single gap below  $B_{c2(\pi)} \simeq \Phi_0/(2\pi\xi_\pi^2)$  with a sharp increase with decreasing field ( $\sigma_\mu \propto 1/b_\pi$ ).

The predicted behavior of  $\sigma_\mu$  is surprisingly close to that found in the earlier reports on  $\text{MgB}_2$ .<sup>4-6</sup> Prior to the establishment of double gap superconductivity, the strong enhancement of  $\sigma_\mu$  at lower fields was attributed to uncontrolled influence of flux pinning.<sup>4,5</sup> Although the flux pinning still remains as a possible cause of enhanced  $\sigma_\mu$ , we argue that the nonlocal effect discussed above may play a significant contribution to the observed field dependence of  $\lambda$ . To visualize the situation, we reproduce the previous data<sup>5</sup> and a result of curve fitting by Eq. (15) with  $B_{c2(\pi)}$  as a free parameter; the data below 0.1 T were excluded from the fit as the field dependence of  $\sigma_\mu$  was reversed probably due to the reduced flux density near the lower critical field. The curve is in excellent agreement with data with  $B_{c2(\pi)} = 0.53(1)$  T [ $\xi_\pi = 25(1)$

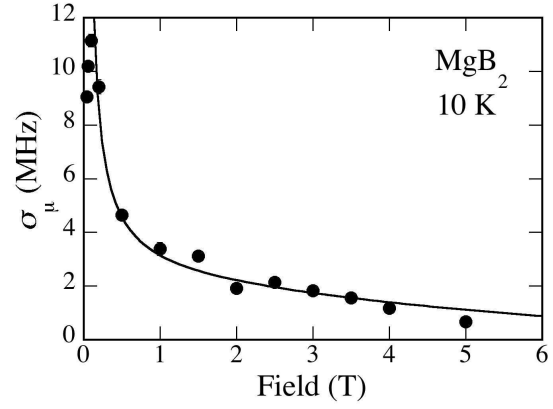


Fig. 1. Muon spin relaxation rate in the superconducting state of  $\text{MgB}_2$  deduced by fits using a Gaussian damping (data reproduced from Ref.<sup>5</sup>). Solid curve is a fit by the model taking account of nonlocal effect arising from double gap structure in the order parameter (see text for detail).

nm], reproducing the tendency of steep upturn with decreasing field below the presumed upper critical field for the smaller gap. The field dependence of  $\sigma_\mu$  reported by other groups exhibit qualitatively similar trend with a sharp upturn of  $\sigma_\mu$  below  $\sim 0.5$  T, and thereby it is likely that the relevant feature stems from an intrinsic property that might be ascribed to the nonlocal effect.

The present discussion may cast some doubts to the earlier model<sup>6</sup> for explaining the behavior of  $\sigma_\mu$  in  $\text{MgB}_2$ ; they assume that  $B(\mathbf{r})$  consists of a weighted sum of the London expression [Eq. (13)] with the cutoff replaced by  $\xi_\pi$  and  $\xi_\sigma$  for the respective components. Unfortunately, the London approximation would certainly fail for the smaller gap over the field range  $b_\pi > 0.25$ .<sup>3</sup>

It must be noted that, while the agreement between data and the present model in Fig. 1 is encouraging, our estimation is no more than a very crude approximation [e.g., the use of  $\bar{A}$  in Eq.(4)] and more precise evaluation is clearly needed. Moreover, there are other sources of nonlocal effects that should be considered as well. For example, recent calculation based on the quasiclassical Eilenberger theory indicates the presence of a nonlocal effect even for the single gap, where the effect may be observed as a gradual increase of  $\lambda$  with increasing field.<sup>7</sup> A similar model for the multigap superconductors is strongly awaited for the detailed understanding of the nonlocal effect.

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